QUANTIZATION OF GRAVITY IN THE THEORY WITH THE MASSIVE VECTOR FIELD

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The quantum theory of gravity is considered based on the assumption that gravitational interaction occurs by means of the vector field of the Planck mass. Gravitational emission is considered as a process of the decay of proton into some matter fields at the Planck scale. Within the framework of grand unification vector field of the Planck mass may be thought of as those which realize the interaction between leptons and quarks.

In the Einstein theory of gravity [1] with the Lagrangian

$$L = -\frac{1}{16\pi G}\sqrt{g}R, \tag{1}$$

gravitational field is defined by the components of the metric tensor $g_{\mu\nu}$. If to take $g_{\mu\nu}$ in the form

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{2}$$

where $\eta_{\mu\nu}$ is the background metric and $h_{\mu\nu}$ is the small perturbations of the background metric, one comes to the theory of the fields $h_{\mu\nu}$ in the background metric. Quantization of gravity reduces to quantization of the field $h_{\mu\nu}$. As follows from the theory [1], the field $h_{\mu\nu}$ is a massless one of the spin 2.

Quantum theory of gravity based on the above approach is nonrenormalizable. According to the dimensions analysis, the nonrenormalizable theories have the negative mass dimensions. The Newton constant G has the dimensions $[\mathrm{m}]^{-2}$ ($\hbar = c = 1$). In the theory of the weak interaction, the Fermi constant G_F also has the dimensions $[\mathrm{m}]^{-2}$. Renormalization of the weak interaction is effected within the framework of the theory with the massive vector boson [2]. From this, it is natural to consider gravity within the framework of the theory with the massive vector boson.

Let us assume that gravitational interaction occurs by means of the vector field, denote it Pl. Let us define the coupling of the theory as

$$g = (\hbar c)^{1/2}. \tag{3}$$

Then the Newton constant is given by

$$G = \frac{g^2}{m_{Pl}^2}, \tag{4}$$

where $m_{Pl} = (\hbar c/G)^{1/2}$ is the Planck mass. This makes natural to assume that the field Pl has the mass equal to the Planck mass. One can develop the theory of gravity with the use of the theory with the massive vector field [2]. To obtain the renormalizable theory, the original field Pl is taken to be massless and acquires mass due to the Higgs mechanism.

According to the standard theory [2], take the Lagrangian of interaction of the matter fields ψ with the massive vector field Pl in the form

$$L = -gJ_\mu(x)Pl_\mu(x). \tag{5}$$

The current J_μ is given by

$$J_\mu(x) = \bar{\psi}(x)O_\mu\psi(x), \tag{6}$$

where O_μ is some matrices. Matrix element of the Lagrangian (5) is of the order

$$M \sim \frac{g^2}{m_{Pl}^2 - q^2}. \tag{7}$$

In the low energy limit $q^2 << m_{Pl}^2$, this takes the form

$$M \sim \frac{g^2}{m_{Pl}^2}. \tag{8}$$

Matrix element (8) describes the processes of the type

$$\psi_1\psi_2 \to Pl \to \psi_1\psi_2, \tag{9}$$

which correspond to the Newton interaction of the fields ψ_1, ψ_2. Also matrix element (8) describes the processes of the type

$$\psi_1\psi_2 \to Pl \to \psi_3\psi_4, \tag{10}$$

which correspond to the transformation of the fields ψ_1, ψ_2 into the fields ψ_3, ψ_4. It is natural to thought of gravitational emission as a process of the type (10). Then gravitational radiation may be identified with some matter fields ψ. In view of the theory with the Lagrangian (5), the Newton potential φ_N and consequently the metric fields $h_{\mu\nu}$ are effective. From this it follows that the metric fields $h_{\mu\nu}$ are not quantized, and the Einstein theory describes only the classical gravity.

The theory of gravity with the massive vector field is consistent with the quantum field theory. This allows to incorporate gravity in the standard scheme of grand unification. Let us consider unification of electromagnetic $U(1)$, weak $SU(2)$ and strong $SU(3)$ interactions within the framework of the $SU(3) \times SU(2) \times U(1)$ model [3]. Usually it is supposed that unification of the interactions leads to the confluence of the electromagnetic α_e, weak α_w and strong α_s couplings into one coupling of grand unification α_{GU}. That is the strong symmetry is described by one simple group, e.g. $SU(5)$. The energy of grand unification $E_{GU} \sim 10^{15}$ GeV is determined as that when the couplings of all the interactions are the same. Let us assume that unification of the interactions is not accompanied with the confluence of the couplings into one coupling. That is the strong symmetry is described by the $SU(3) \times SU(2) \times U(1)$ group. Suppose that the energy of grand unification is the Planck mass $E_{GU} \sim m_{Pl} \sim 10^{19}$ GeV. In this case we can identified the field Pl with the fields X, Y which realize interaction between quarks and leptons. Then the Lagrangian (5) describes interaction of quarks and leptons with the fields X, Y, with the current J_μ corresponds to the elementary processes of the type

$$ud \to X \to e^+\bar{u}, \quad ud \to Y \to \bar{d}\bar{\nu}_e, \quad ud \to Y \to e^+\bar{u}. \tag{11}$$

These lead to the reactions of the decay of proton

$$p \to e^+\pi^0, \ \pi^+\bar{\nu}_e, \ e^+\pi^-\pi^+. \tag{12}$$

From this gravitational emission may be identified with the process of the decay of proton into leptons at the Planck scale. Then gravitational radiation may be identified with neutrinos ν.

References

[1] I.L. Buchbinder, S.D. Odintsov, I.L. Shapiro, Effective action in quantum gravity (Bristol and Philadelphia: IOP Publishing Ltd, 1992)

[2] N.N. Bogoliubov and D.V. Shirkov, Quantum fields, 2nd Ed. (Nauka, Moscow, 1993, in Russian)

[3] L.B. Okun, Leptons and quarks (Nauka, Moscow, 1981, in Russian)